

523

NAVAL POSTGRADUATE SCHOOL

Monterey, California



ESTIMATING SEARCH EFFECTIVENESS WITH LIMITED INFORMATION

R. N. Forrest
//

September 1993

Approved for public release; distribution is unlimited.

Prepared for:
Naval Research Project Office
Vienna, VA 22180

FOI LMS
E-02 1012 NPS-440-73-001

**NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA**

**Rear Admiral T. A. Mercer
Superintendent**

**Harrison Shull
Provost**

This report was prepared for and funded by the Naval Research Project Office.

Reproduction of all or part of this report is authorized.

This report was prepared by:

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE

September 1993

3. REPORT TYPE AND DATES COVERED

Technical Jul 93 - Sep 93

4. TITLE AND SUBTITLE

Estimating Search Effectiveness with Limited Information

5. FUNDING NUMBERS

6. AUTHOR(S)

R. N. Forrest

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Postgraduate School
Monterey, CA 93943

8. PERFORMING ORGANIZATION
REPORT NUMBER

NPS-AW-93-001

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Naval Research Project Office
P.O. Box 1456
Vienna, VA 22180

10. SPONSORING / MONITORING
AGENCY REPORT NUMBER

N4175693WR39149

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The report contains descriptions of some models that can be used to estimate the effectiveness of detection systems about which there is limited information. Examples are included of the use of the models for this purpose.

14. SUBJECT TERMS

Search Modeling, Detection Modeling, Localization Modeling,
Classification Modeling

15. NUMBER OF PAGES

26

16. PRICE CODE

17. SECURITY CLASSIFICATION
OF REPORT

Unclassified

18. SECURITY CLASSIFICATION
OF THIS PAGE

Unclassified

19. SECURITY CLASSIFICATION
OF ABSTRACT

Unclassified

20. LIMITATION OF ABSTRACT

UL

Table of Contents

I.	Introduction.....	1
II.	Lateral Range Curves and Sweep Width.....	2
III.	The Area and Barrier Search Models.....	4
IV.	The Use of the Models to Estimate Detection System Effectiveness.....	7
V.	Localization Effectiveness.....	10
VI.	Classification Effectiveness.....	13
	Appendix 1. A Wake Detection Model.....	14
	Appendix 2. Average Relative Speed in a Series of Straight Line Encounters.....	16
	Appendix 3. A Classification Model.....	18
	Appendix 4. A Comparison of Two Classification Models.....	19
	References.....	21

I. Introduction

This report contains descriptions of models that can be used to estimate the search effectiveness of detection systems whose characteristics are not well defined. In addition, the report contains example analyses that illustrate their use. The analyses ignore critical factors such as the cost associated with a detection system in establishing and maintaining a search. For this reason, they illustrate, at best, preliminary analyses.

Because the focus of the report is on detection systems whose characteristics are not well defined, only models that require limited information are considered in the analyses. For example, definite range law detection models and random search models that are described in Reference 1 and elsewhere.

A general definite range law detection model describes a hypothetical detection system that detects a target with probability p if the target's range is less than or equal to a definite range and with probability zero if the target's range is greater than the definite range. In the example analyses in this report, p is set equal to one which is consistent with the classical definite range law of Reference 1.

A random search model describes a search that has a diminishing returns characteristic that can be attributed to errors in implementing a search plan. In addition to requiring limited information, random search models can be used to describe barrier search plans as well as area search plans.

In the example analyses, the primary measure of effectiveness for a detection system that is used for area search is the time to first detection of a target in the search region. And, the primary measure of effectiveness for a detection system that is used for barrier search is the probability of detection given that a target crosses the barrier.

The localization area associated with a detection and the time required to classify false targets are additional measure of a detection system's effectiveness. Both localization models and classification models are discussed in the report.

II. Lateral Range Curves and Sweep Width

A lateral range curve is a plot of the probability that a target will be detected in a straight line encounter as a function of the target's horizontal range at its closest point of approach (CPA). A straight line encounter is an encounter in which a target's track is a straight line relative to a detection system's sensor. (As defined here, an encounter between a detection system and a target occurs while and only while the probability of detection for an observation is greater than the false alarm probability for the observation.) Two lateral range curves are shown in Figure 1.

Sweep width W is defined in Reference 1 as the area under a target's lateral range curve. It is defined more operationally but more narrowly in Reference 2 for lateral range curves that are symmetric about zero CPA by: $W = 2 \cdot r$ where r is the lateral range such that the probability of detecting a target given that the target's lateral range is greater than r is equal to that of not detecting the target given the target's lateral range is less than or equal to r .

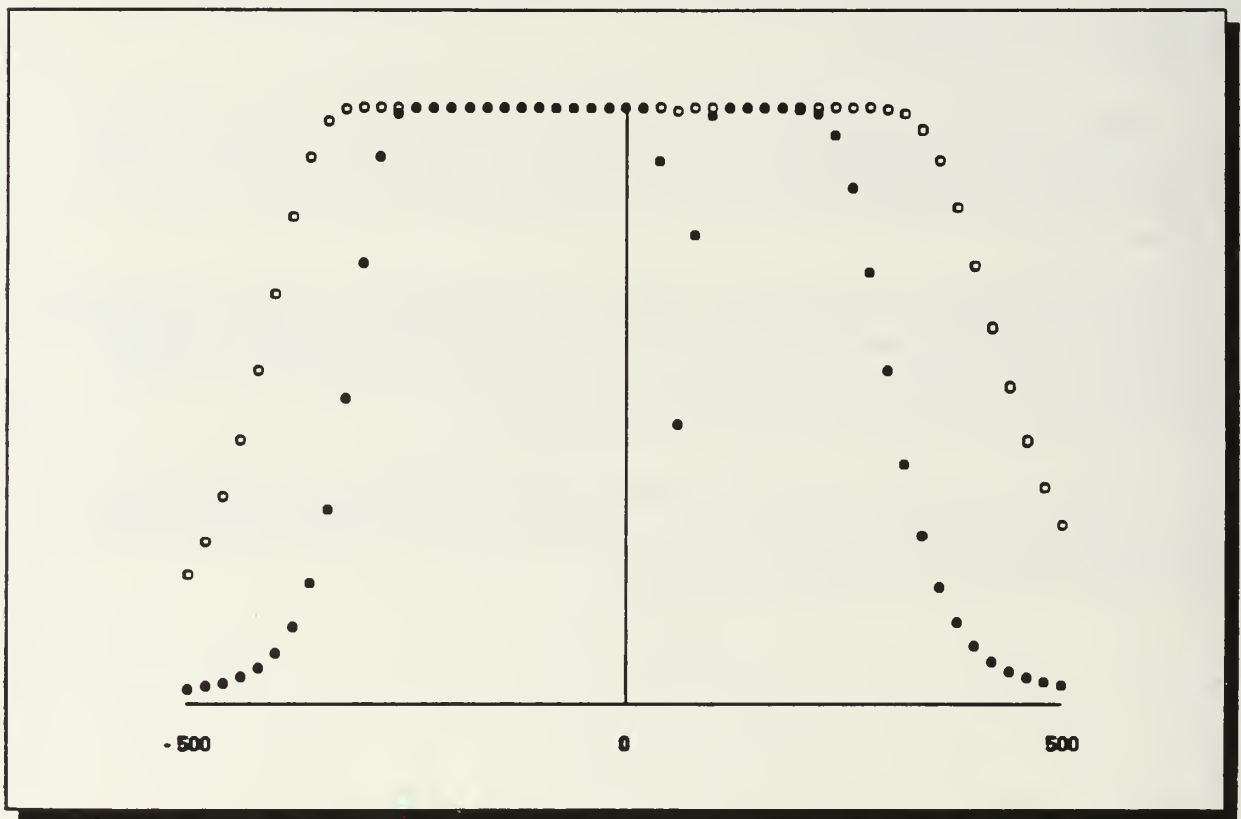


Figure 1. Two lateral range curves for a MAD detection system in a straight line encounter with a magnetic dipole target. The curve defined by the open circles is for a crosscorrelation detector. The remaining curve is for an energy detector. The curves are based on an encounter model that is described in Reference 3. The horizontal axis gives the target's horizontal range at CPA in meters and, for both curves, at CPA the vertical axis is 1.

As an illustration, in Figure 1, the crosscorrelation detector lateral range curve suggests a definite range value of 400 meters and the energy detector lateral range curve suggests a definite range value of 350 meters. Although neither curve is symmetric about CPA, for the purposes of this report, they both satisfactorily approximate the description of a lateral range curve for a definite range law detection system.

Both the formal definition of sweep width in Reference 2 and the more operational definition in Reference 3 imply that the sweep width W against a target for a definite range law detection system with a definite range R is given by: $W = 2 \cdot R$. Using this result, the definite range law approximations that are suggested above give a sweep width of 800 meters for the crosscorrelation detector and a sweep width of 700 meters for the energy detector against the MAD target to which the two lateral range curves refer.

In Appendix 1, it is shown that a sweep width can be associated with a wake (line) as well as a vessel (point). The association is based on a model that is described in Reference 4.

If a lateral range curve cannot be determined for a detection system, but an estimate of the detection system's maximum detection range for a target can be, then by using a definite range law model with the estimate as the definite range, the sweep width for target is twice the definite range. If the detection system represents a threat and the estimate is not less than the true value, then this would be a conservative procedure.

III. The Area and Barrier Search Models

The first random search model that is considered describes an area search in which either a target or a detection system's sensor or both move randomly through a search region. The model gives the probability that the time to first detection of the target by the detection system is at or before a specified time t as:

$$(1) \quad P(T \leq t) = 1 - \exp[-w_A \cdot W \cdot t / A]$$

where, T is the time to first detection, the primary measure of effectiveness, w_A is the average speed of the target relative to the sensor, W is the detection system's sweep width against the target and A is the area of the search region. A development of the model is given in Reference 1 and an alternative development is given in Reference 5. The determination of an average relative speed w_A is described in Appendix 2. And, as indicated in Appendix 2, for the analyses in this report, w_A can be satisfactorily approximated by either the sensor's speed v or the target's speed u .

The model that is used to describe both ladder search plans and barrier search plans that are ladder search plans in the target's reference frame gives the probability P that a target will be detected in a search as:

$$(2) \quad P = W/s \quad \text{for} \quad W/s \leq 1.$$

Here, W is the definite range law detection system's sweep width against the target and s is the ladder search track spacing.

In a conservative version of the model, overlaps and holidays in the search coverage are considered and P is given by:

$$(3) \quad P = 1 - \exp[-W/s].$$

In this version, the restriction on W/s can be relaxed.

Both models are developed in Reference 1. The requirements for establishing a barrier search that is a ladder search in a target's reference frame are given in Reference 5 where the model described by Equation 3 is referred to as a degraded ladder search model.

A model that has been used to model back-and-forth barrier search plans gives the probability P that a target that crosses the barrier will be detected as:

$$(4) \quad P = [W/L] \cdot [1 + v^2/u^2]^{1/2} \quad \text{for} \quad [W/L] \cdot [1 + v^2/u^2]^{1/2} \leq 1.$$

Here, W is the detection system's sweep width against the target, L is the barrier width, u is the target's speed when it crosses the barrier and v is the searcher's speed back-and-forth on the barrier.

If a target's wake is the target of a detection system and a wake element is considered a definite range law target, then it is shown in Appendix 1 that the detection system's sweep width W_w against the target is given by:

$$(5) \quad W_w = R + 2 \cdot L_w / \pi$$

where R is the detection system's definite range against a wake element and L_w is the wake length. A somewhat more detailed model is described in Reference 4.

A model that is developed in Reference 5 describes a detection system that detects intermittent signals of length δt . The model has the following characteristics: A target generates signals at random times such that the time between the end of one signal and the beginning of the next signal is a random variable that is determined by a Poisson process. A signal is detected if and only if the signal is being generated when the target is at a range that is less than or equal to a definite range (a definite range law approximation is used). With this model, the lateral range curve for a target is given by:

$$(6) \quad p(x) = 1 - \exp \{ -[2 / (w \cdot \tau)] \cdot (R^2 - x^2)^{1/2} + \delta t / \tau \} \quad \text{for } |x| \leq R$$

where w is the speed of the target relative to the detection system's sensor, τ is the average time between signals and R is the detection system's definite range. The detection system's sweep width W_1 against the target is the integral of $p(x)$ over the interval $-R \leq x \leq R$. Since the integral cannot be evaluated in terms of elementary functions, an approximation based on the lateral range curve for an intermittent target signal that is periodic is considered next.

If the signals are periodic rather than random, the signal period is τ and the signal duration is $\delta t < \tau$, then the lateral range curve is given by:

$$(7) \quad p(x) = [2 / (w \cdot \tau)] \cdot (R^2 - x^2)^{1/2} + \delta t / \tau \quad \text{for } R < w \cdot (\tau - \delta t) / 2.$$

In this case, the integral of $p(x)$ over the interval $-R \leq x \leq R$ gives the sweep width as:

$$(8) \quad W_1 = \pi \cdot R^2 / (w \cdot \tau) + 2 \cdot R \cdot \delta t / \tau \quad \text{for } R < w \cdot (\tau - \delta t) / 2.$$

where, by definition, W_1 is the detection system's sweep width against the target.

If $[2 / (w \cdot \tau)] \cdot (R^2 - x^2)^{1/2} < .1$, the lateral range curve that is defined by Equation 6 can be approximated by:

$$(9) \quad p(x) = [2 / (w \cdot \tau)] \cdot (R^2 - x^2)^{1/2} \quad \text{for } R \leq |x|$$

and, therefore, from Equation 7 and Equation 8, the sweep width for the random intermittent target model can be approximated by:

$$(10) \quad W_1 = \pi \cdot R^2 / (w \cdot \tau) \quad \text{for } [2 / (w \cdot \tau)] \cdot (R^2 - x^2)^{1/2} < .1 \text{ and } \delta t \ll \tau.$$

This approximation is used in Section IV to estimate the effectiveness of a hypothetical detection system that detects intermittent target signals.

As a basis for comparing the effectiveness of the hypothetical detection system that is analyzed in Section IV, a modified random search model is used here to estimate the effectiveness of a detection system that monitors a sensor field. In this model, a target moves randomly with speed u through a field of n definite range law sensors. For this case, $w = u$ and the probability that the time to first detection of a target is at or before a specified time t is given by:

$$(10) \quad P(T \leq t) = 1 - \exp \{ [(-u \cdot t) / A] \cdot \sum_i^n W_i \}$$

Here, T is the time to first detection by a sensor, W_i is the detection system's sweep width against the target for the i th sensor and A is the area covered by the field.

In Reference 1, a detection system's sweep rate S against a target is defined by the relation: $S = w \cdot W$ where W is the detection system's sweep against the target and w is the target's speed relative to the detection system's search sensor. Clearly, A / S is a measure of the time to search a region of area A by a detection system with a search rate S . However, in general, $P(T \leq A / S) < 1$, since the ratio does not account for a number of factors that determine the time to first detection. In particular, it does not account for overlaps and holidays in the coverage of a searched region.

The random search model can be considered to account for overlaps and holidays in a conservative way. And, for the random search model, the expected or average time to detect a target is given by:

$$(11) \quad E(T) = A / S.$$

And, from Equation 1,

$$(12) \quad P(T \leq A / S) = .63.$$

IV. The Use of the Models to Estimate Detection System Effectiveness

Three analyses of the effectiveness of a hypothetical detection system are given in this section to illustrate the use of the models that are defined above. In the analyses, use is made of a random search model, a degraded ladder search model and a back-and-forth barrier search model.

The analyses are based on a fictitious scenario that is defined as follows: Intelligence information suggests that a detection system exists that could represent a threat to submarines in certain operations. The information indicates that the system is used on an aircraft that conducts searches in operating areas. The information also indicates that the detection system may only detect intermittent signals that are generated at random times. The information does not indicate the nature of the signals nor the mechanism by which they are generated. However, it does suggest the following values: a 120 nautical mile by 200 nautical mile rectangular search region, an aircraft search speed v equal to 200 knots, a target speed u equal to 5 knots, a definite range law detection system with a definite range equal to 20 nautical miles, an intermittent signal with an average time between signals equal to 2 hours and signal duration t equal to 2 minutes and a ladder search plan with a track spacing equal to 40 nautical miles. (The track spacing value is not consistent with the assumed nature of the signals; but, since the signal information and the track spacing value come from different sources, the inconsistency is accepted.) The information suggests a barrier front of 100 nautical miles when the system is used in a barrier search.

The First Analysis:

A random search model is used in the first analysis to estimate the detection system's effectiveness when it is used with an area search plan. The scenario values and Equation 9 give a sweep width $W_1 = 3.14$ nautical miles for a definite range law detection system against a target that generates random signals that are determined by a Poisson process. The sweep rate against the target $S_1 = 628$ square nautical miles per hour. Since $A = 24,000$ square nautical miles, by Equation 11, the expected time to detect the target $E(T) = 38.2$ hours.

The Second Analysis:

A degraded ladder search model is used in the second analysis to estimate the detection system's effectiveness with an area search plan. The analysis also applies to a barrier search plan that is a ladder search in a target's reference frame when the target's course and speed are assumed to be known. For the scenario track spacing, $s = 40$ nautical miles and the sweep width $W = 3.14$ nautical miles, Equation 3 gives a probability of detection $P = .0755$.

If n independent degraded ladder searches can be made, the probability of detection is given by:

$$(13) \quad P = 1 - \exp(-n \cdot W/s)$$

From Equation 13, the number of degraded ladder searches n required to have a probability of detection equal to a specified value P can be determined from the relation:

$$(14) \quad n = - (s / W) \cdot \ln (1 - P)$$

For $P = .63$, $n = 12.66$, so 13 degraded ladder searches give a probability of detection comparable to the probability of detection for a random search of 31.8 hours. The length of an ideal ladder search leg is 200 nautical miles. Since a ladder search of 3 legs covers the search region, the length of an ideal ladder search is 680 nautical miles including the 40 nautical miles between the legs. So, 13 ladder searches would require approximately 33.8 hours to complete, since each ladder search requires 2.6 hours.

The above result is not surprising based on the following argument from Reference 1: With m the number of legs and b the length of a leg, the ladder search area $A = m \cdot s \cdot b$ and, ignoring the time to move between legs, the time to complete a ladder search $t = m \cdot b / u$. Then substituting these values into Equation 1 gives Equation 3 that defines the degraded ladder search model since $w_A = u$ in this case.

If the ladder search model had been used rather than the degraded ladder search model, then, Equation 2 gives $P = .0785$. If n independent ladder searches can be made, the probability of detection is given by:

$$(15) \quad P = 1 - (1 - W / s)^n \quad \text{for } W / s \leq 1.$$

From Equation 15, the number n of ladder searches required to have a probability of detection equal to a specified value P can be determined from the relation:

$$(16) \quad n = \ln (1 - P) / \ln (1 - W / s)$$

For $P = .63$, $n = 12.15$, so again 13 ladder searches give a probability of detection comparable to the probability of detection for a random search of 31.8 hours. This is not surprising, since $W/s = .0785$ and, from Equation 2 and Equation 3, the difference between the ladder search model and the degraded ladder search model is negligible for $W / s < .1$.

The Third Analysis:

A back-and-forth barrier search plan is not consistent with the scenario. However, a stationary barrier search plan that is a ladder search in the target's reference frame is consistent with the scenario. Since a ladder search barrier plan requires a specific target course, a value for the target's course is assumed to be specified by the geography of a strait. With this modified scenario, $P = .0755$ using the degraded ladder search model.

Clearly, a barrier with a detection probability of .0755 would not be satisfactory for most operations. The difficulty arises because of the track spacing which is appropriate for a continuous target signal but not for the intermittent target signal that is specified by the scenario.

For a continuous target signal and a definite range law detection system, the barrier model gives a probability of detection equal to 1. This suggests a more simplistic barrier analysis: Since the time to complete a leg ladder search in the relative space of the target is approximately .5 hours from Reference 5, the probability that at least one target signal will be generated in .5 hours is approximately 0.22. And the probability that the target will be at a range from the detection system's sensor that is less than or equal to 20 nautical miles is approximately .4. So, the probability that a target that crosses the barrier will be detected is approximately .088.

V. Localization Effectiveness

As a basis for determining the evaluating the effectiveness of the hypothetical detection system of Section IV, consider a detection system that employs a sensor field of n sensors, for example, a field of n magnetometers. Suppose the field covers the search region of the scenario of Section IV and suppose the magnetometers are definite range law sensors, each with a definite range $R = .5$ nautical miles against the target. Since $W = 2R$ for each sensor, Equation 10 becomes:

$$(17) \quad P(T \leq t) = 1 - \exp \{-2 n \cdot u \cdot t R / A\}$$

And, the expected or average time for a target to be detected is given by:

$$(18) \quad E(T) = A / (2 \cdot n \cdot u \cdot R)$$

So, for the hypothetical detection system in an area search described by the random search model in the first analysis of Section IV, the number of magnetometers required to give a comparable expected time to detection can be determined with the relation:

$$(19) \quad n = A / [2 \cdot u \cdot R \cdot (T)]$$

For the values being considered, $n = 125.6$. So a field of 126 sensor, each with a definite range of .5 nautical miles, would have a comparable expected times to detection. However, when a target is detected by a magnetometer in the field, it is localized within a circle whose radius is .5 nautical miles and when it is detected by the detection system according to the above analysis, it is localized within a circle whose radius is 20 nautical miles.

Suppose the information in the scenario of Section IV had included a localization capability for the hypothetical system that was specified as follows: When a detection occurs, the final localization is conducted with a definite range law detection system with a definite range against the target of .5 nautical miles. Since the area of the localization disk for the hypothetical detection system is 1,2567 square nautical miles, by Equation 19, the expected time to achieve final localization would be 6.3 hours.

Suppose, in addition, that the information available suggested that a target's bearing at detection is localized to an interval of 40 degrees. Then, at detection, a target would be localized to a sector of a circular disk with an area of 140 square nautical miles, and, by Equation 19, the expected time to achieve final localization would be .7 hours.

As an alternative to the scenario of Section IV, suppose the information had indicated the existence of detection system with several sensors that detect target signals. And, the information indicated that, on detection, a sensor determined with reasonable accuracy a target's bearing but not its range. In addition, suppose that the information suggested the following values: a bearing error of ± 5 degrees and a sensor separation of 10 nautical miles. With this scenario, a localization capability as a function of target range could be estimated by noting the area of the region determined by intersecting bearing lines from sensor locations separated by

10 nautical miles. If the ranges considered are less than 500 nautical miles, a flat earth model should be sufficient for the model geometry. In this model, bearing errors are symmetric about the true bearing and are no greater than the specified limits. Figure 2 shows a localization region generated by bearing observations from two sensors. If more than two sensor stations are involved, the localization region is not immediately evident. In this case, the limiting bearing lines for each sensor pair define a localization region, so, for n sensors, the number of localization regions is equal to the number of combinations of n things taken 2 at a time.

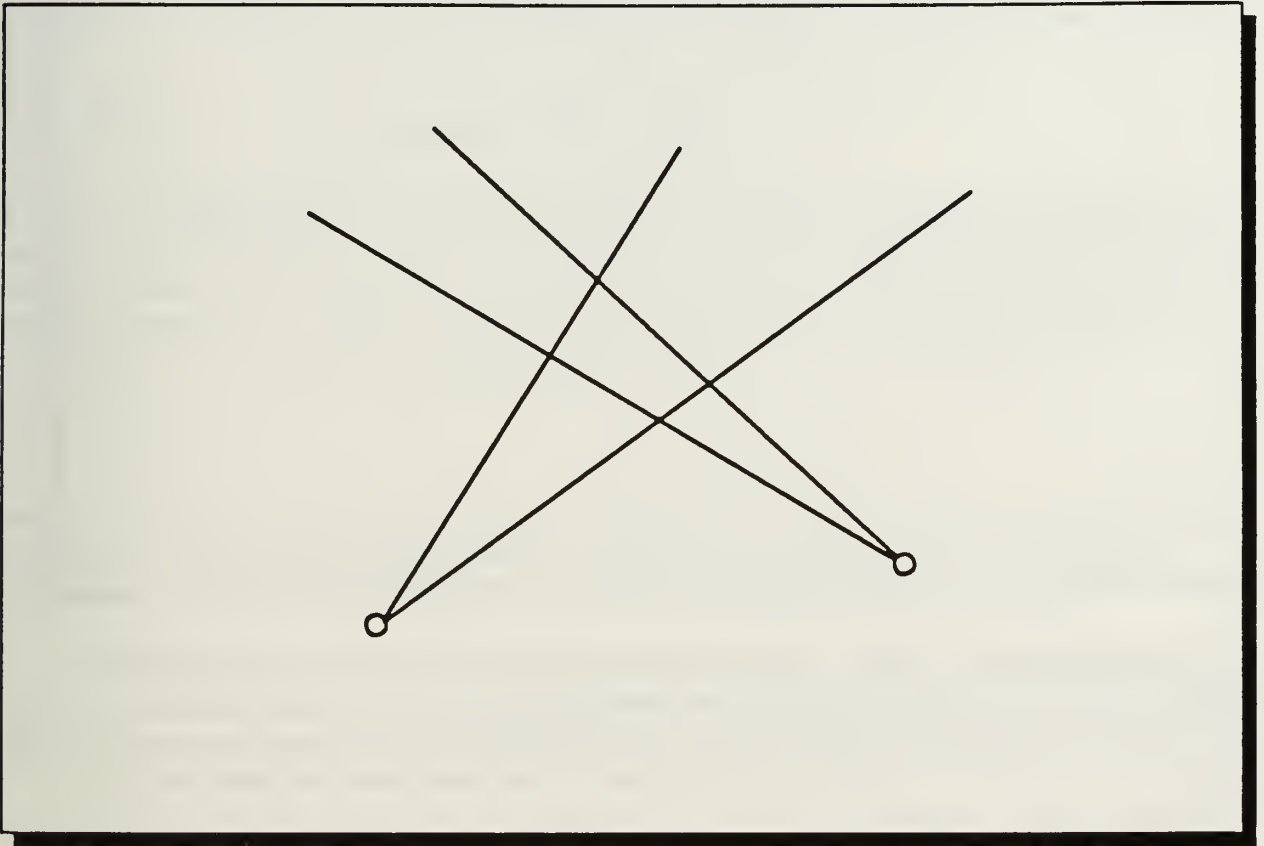


Figure 2. A map showing the location of two sensor stations. The sensors are located at the circled points and the lines radiating from the points represent the limiting bearing lines for a detected target. The localization region is the region bounded by the four bearing lines.

A model is described in Reference 6 which eliminates this problem. It is an example of a bearings only localization model. The model determines a bivariate normal probability distribution for the location of a target based on bearing errors that are normally distributed. A localization region is determined by specifying a required probability of containment, typically .86 (a two sigma region), and the localization regions that are generated by the model are bounded by ellipses. Figure 3 shows a localization region generated by three sensor station. A similar model and models that determine elliptical localization regions for an array of sensors based on range errors as well as bearing errors are described in Reference 7.

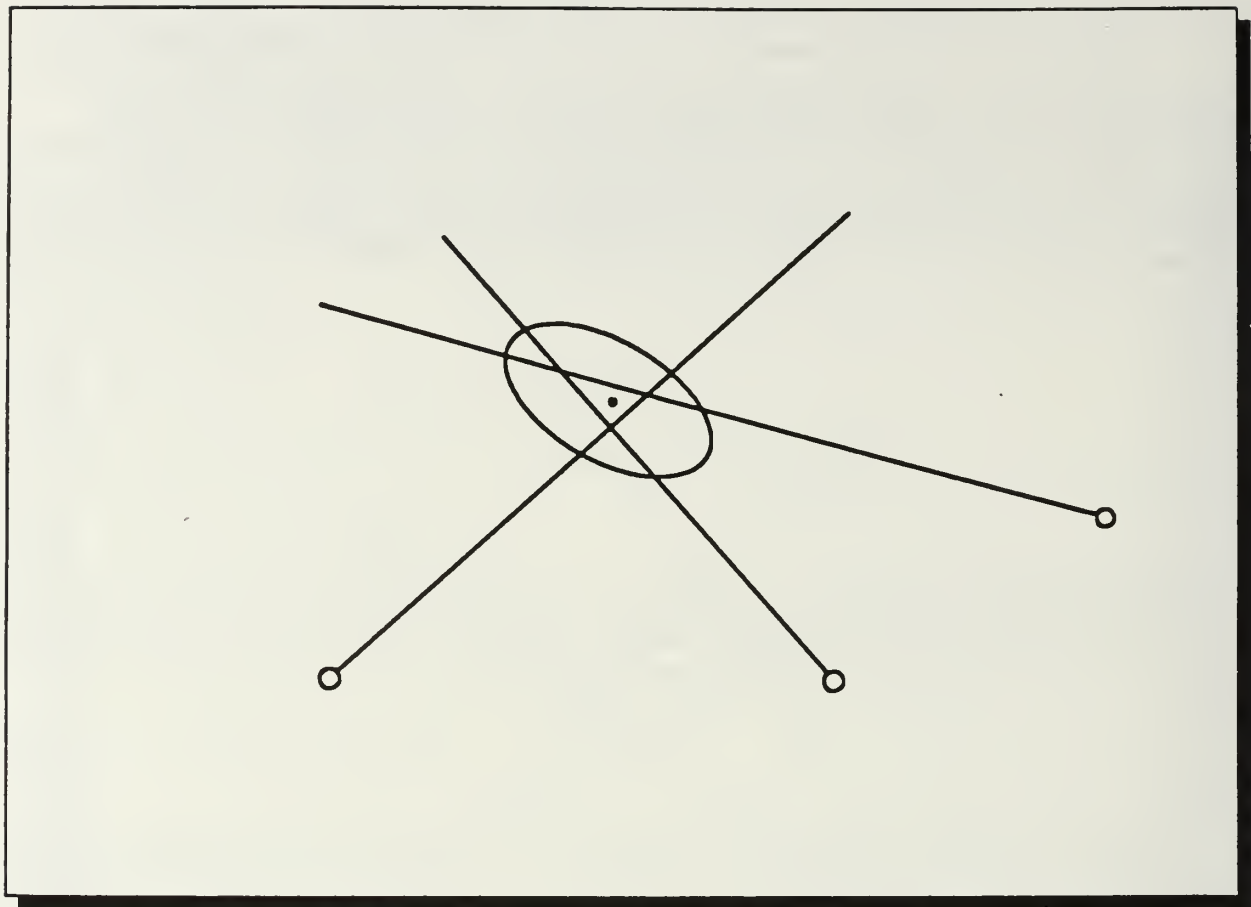


Figure 3. An example of an elliptical localization region generated by the model described in Reference 6. The dimensions of the ellipse are determined by the required probability of containment for the localization. The point within the elliptical boundary is at the estimated position of the target and the region within the ellipse is a minimum area confidence region for the estimate.

VL Classification Effectiveness

The analyses in Section IV do not address a detection system's classification effectiveness. A model that does this is discussed here. The model which is developed in Appendix 3 determines classification effectiveness through the time to first detection.

The model describes an area search in which a searcher moves randomly through a search region that contains both a target and false targets. The model is based on the random search model and it gives the probability that a target is detected at or before a specified time t as:

$$(20) \quad P(T \leq t) = 1 - \exp [-(w_A \cdot t \cdot W_F) / A]$$

where W_F can be considered to be the detection system's sweep width against the target in the presence of false targets and, as in the random search model, T is the time to first detection, the primary measure of effectiveness, w_A is the average speed of the target relative to the detection system's sensor and A is the area of the search region. The sweep width in the presence of false targets W_F is related to the sweep width W by:

$$(21) \quad W_F = W / (1 + \tau \cdot v)$$

where t is the time searching plus the time classifying, τ is the average time to classify a false target and v is the average number of false target detections per unit of time while not classifying.

With false targets, the ratio of the expected time to detect a target when there are false targets to that when there are no false targets is: W / W_F and this is a measure of the reduced effectiveness of a detection system that results from its detection of false targets.

As an example, suppose $\tau = 15$ minutes and $v = .3$ false targets per hour. Then, for the first analysis in Section IV, the effective sweep width $W_F = .81$ nautical miles rather than 3.14 nautical miles. So, $W / W_F = 3.87$ and the expected time to detect the target is equal to 148.1 hours rather than 38.2 hours. And, in addition, the probability the target will be detected in 38.2 hours is .23 rather than .63.

Like the random search model, this classification model is transparent, that is, in addition to the effect of the other parameters, the effect of the parameters τ and v is evident. But, the information required to use the model is significantly greater than that required for the random search model.

A second type of classification model is discussed in Appendix 4. Although the model describes a sensor that moves relative to a target and false targets whose relative positions remain fixed, it does illustrate what in general is a simulation model's increased versatility as well as increased information requirement relative to the "back-of-the-envelope" analytical models of this report.

Appendix 1. A Wake Detection Model

The wake detection model is based on the following assumptions: A target wake is a moving straight line in a horizontal plane. The wake is detected during an encounter if and only if the horizontal range of a detection system's sensor from at least one point on the wake is equal to or less than a definite value. In a straight line encounter between a detection system's sensor and a wake, the angle between the projection of the sensor's track on the plane of the wake and the wake is a random variable that is equally likely to have any value between 0° and 90° . The encounter geometry for a straight line encounter between a wake and a detection system's sensor is shown in Figure 4.

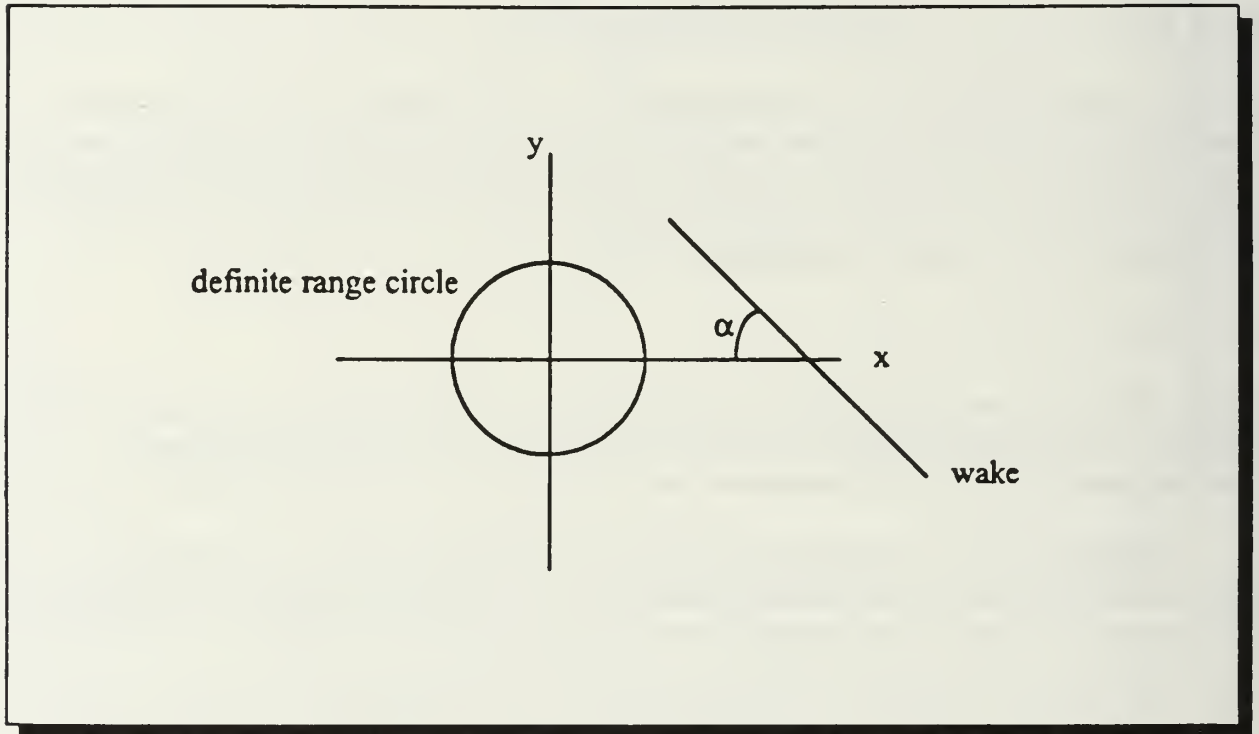


Figure 4. The encounter geometry in the plane of the wake for a straight line encounter between a wake and a detection system's sensor. The angle between the wake and the sensor's projected track on the plane of the wake is α . The horizontal distance between the midpoint of the wake and the sensor's track is x and the encounter geometry is at the time the midpoint of the wake is at CPA.

A lateral range curve that is generated with the model can be used to determine the sweep width of a detection system against a wake for a wake and detection system that can be satisfactorily represented by the model. The sweep width can be determined more directly by first determining the sweep width as a function of α . With x the horizontal range of the midpoint of the wake, the sweep width for a wake of length λ is given by:

$$W_w(\alpha) = 2 \cdot [R + (\lambda / 2) \cdot \cos \alpha]$$

Since $W_w(\alpha)$ is the sweep width given the encounter angle is α and since α is uniformly distributed between 0° and 90° , the sweep width is determined by:

$$W_w = (2 / \pi) \int_0^{\pi/2} W_w(\alpha) \cdot d\alpha .$$

And,

$$W_w = 2 \cdot (R + \lambda / \pi) .$$

Appendix 2. Average Relative Speed in a Series of Straight Line Encounters

A target's speed w relative to a detection system's sensor is a factor in determining the probability the target will be detected in a straight line encounter. This implies that both the lateral range curve and the sweep width are functions of w . The velocity of the target relative to a sensor is given by: the vector equation: $\mathbf{w} = \mathbf{v} - \mathbf{u}$ where \mathbf{w} is the relative velocity, \mathbf{v} is the target's velocity and \mathbf{u} is the sensor's velocity. In the encounters that are considered in this report, the velocity vectors can be considered to be coplanar, and, with ϕ the angle between \mathbf{v} and \mathbf{u} , the relative speed is given by: $w = [u^2 + v^2 - 2 \cdot u \cdot v \cdot \cos \phi]^{1/2}$ where u , v and w are the magnitudes of the vectors, that is the sensor's speed, the target's speed and the target's speed relative to the sensor.

If the angle ϕ is assumed to be the value of a random variable that is uniformly distributed between 0° and 360° for each encounter during a search, then an average value for w in the search is given by:

$$w_A = \frac{1}{2\pi} \int_0^{2\pi} w \cdot d\phi$$

The assumptions on which this average is based is consistent with the random search model. However, the improvement in an evaluation of a detection system that used this average with a random search model rather than u or v in evaluating a detection system would likely be negligible unless the speed of the detection system's sensor and the speed of the target were comparable. In particular, the difference between u and w_A if $u/v < .5$ or between v and w_A if $v/u < .5$, should not be significant for the purposes of this report. This can be seen from Figure 5 below which shows a graph of $(w_A - u)/u$ as a function of v/u if $u > v$ or a graph of $(w_A - v)/v$ as a function of u/v if $v > u$. The graph illustrates the small difference between w_A and u for $u/v < .5$ and between w_A and v for $v/u < .5$.

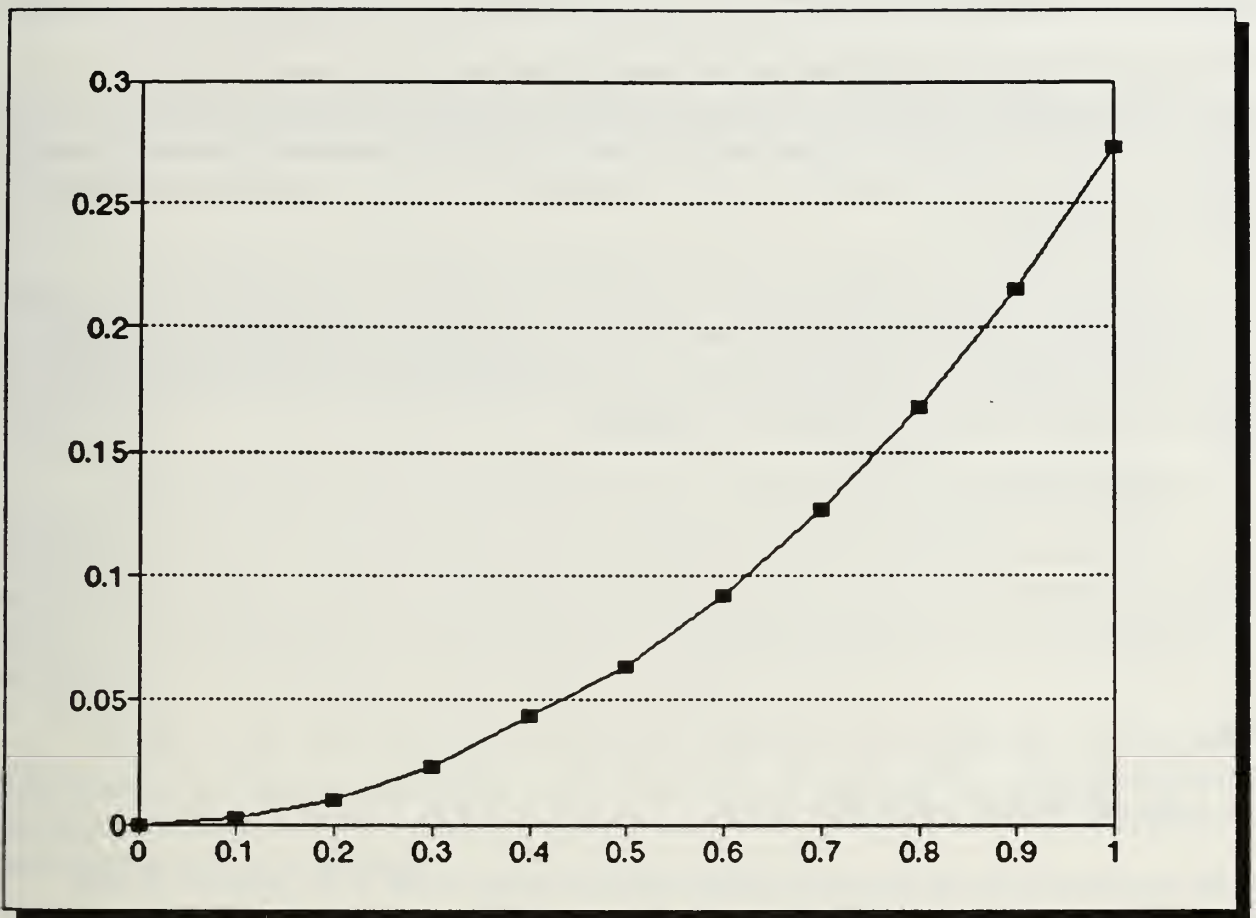


Figure 5. A graph of $(w_A - u)/u$ as a function of v/u if $u > v$ or of $(w_A - v)/v$ as a function of u/v if $v > u$.

Appendix 3. A Classification Model

The model to estimate the effect of classification on the effectiveness of a detection system is described in Reference 8. The model is based on the following assumptions: A search is a random search. Detections cannot be made during the time a contact is being classified. While not classifying, false targets are detected at a rate ν . The average time to classify a false target is τ .

With these assumptions, $t_c = (t - t_c) \cdot \nu \cdot \tau$ where t is the search time and t_c is the time classifying. And, $t - t_c = t / (1 + \nu \cdot \tau)$ is the time during which detections can occur. Hence, based on the random search model, the probability that a target is detected in a search at or before searching for a time t is given by:

$$P(T \leq t) = 1 - \exp [w \cdot W \cdot (t - t_c) / A]$$

or

$$P(T \leq t) = 1 - \exp [w \cdot W_f \cdot t / A]$$

where

$$W_f = W / (1 + \nu \cdot \tau)$$

can be considered to be the detection system's effective sweep width in the presence of false targets.

Appendix 4. A Comparison of Two Classification Models

A comparison is made here between the analytical model that is developed in Appendix 3 and the model that is implemented through a simulation that is described in Reference 6. The purpose of the comparison is to indicate how the limitations of the analytical model might be examined.

The simulation model describes both detection and classification in terms of definite range laws. In the model, the number of false targets in a search region is specified. However, the false targets are identical in their detection and classification characteristics. In this sense, the model is the same as the model of Appendix 3. After a false target is detected, the false target's identity and location is maintained, that is, it is not redetected.

A major difference between the models is in the average false target detection rate: In the simulation model, the average false target detection rate does not change while in the simulation model, as a search continues, the average false target rate trends towards zero. In addition, the simulation allows the characteristics of the false targets to be expressed in terms of the number of false targets in a search region, their average detection and classification ranges and the target's average detection and classification range.

The simulation is limited in that it describes a sensor that moves relative to a target and false targets whose relative positions are fixed. The model of Appendix 3 is more general in that the false target detection rate on which it is based can refer to false targets that move relative to each other as well as to the target.

A simulation model with more flexibility could allow a more penetrating examination of the analytical model. But, for the purpose of this report, the simulation model that is described in Reference 6 does suggest some advantages that might a simulation might have relative to the model of Appendix 3.

The program that implements the simulation model can be used to generate histograms that indicate the distribution of the time to first detection as a function of the number of false targets in a search area, the target's detection range and classification range and the false targets' detection range and classification range. By comparing the histogram for the time to first detection with zero false targets to those for various numbers of false targets, an estimate of a detection system's classification effectiveness can be made. A simulated search track that was generated by the model is shown in Figure 6.

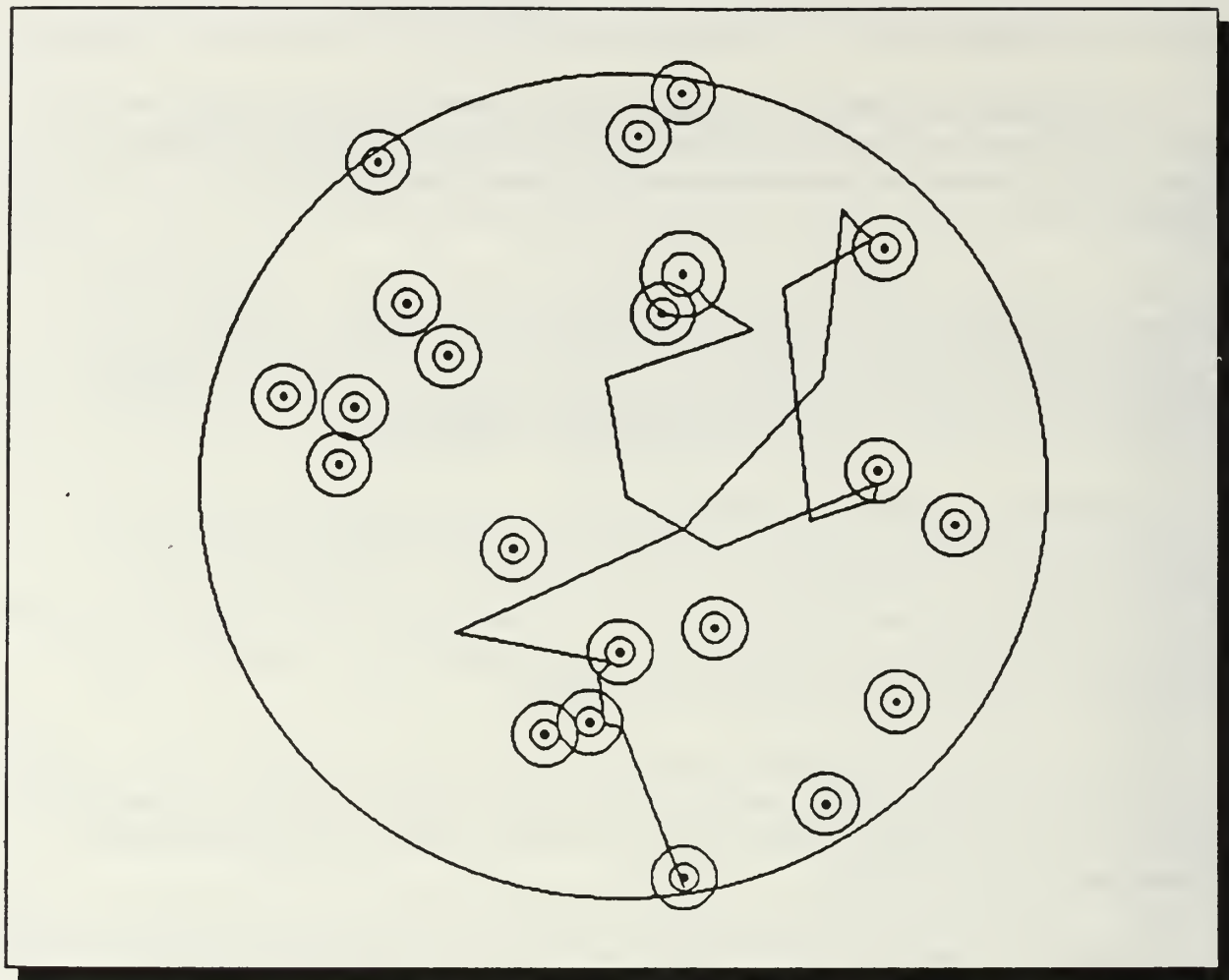


Figure 6. A map generated by the simulation model that is described in Reference 6 showing the track of a detection system's sensor in a random search. The points within the smaller concentric circles represent the location of false targets. The point within the larger concentric circle represent the location of a target. The radius of the inner concentric circles corresponds to a detection system's classification range and the radius of the outer concentric circle corresponds to its detection range.

References

1. Koopman, B. O. , "Search and Screening," OEG Report No. 56, Operations Evaluation Group, Office of the Chief of Naval Operations, Navy Department, Washington, D.C., 1946.
2. "National Search and Rescue Manual," NWP-37(B) or CG 308 1, July 1973.
3. Forrest, R. N., "A Program to Compute Magnetic Anomaly Detection Probabilities," NPS71-88-001 (Second Revision), Naval Postgraduate School, Monterey, CA 93943-5000, October 1988 (Revised March 1990).
4. Forrest, R. N., "Submarine Search Models," NPS55Fo74121 (Revised), Naval Postgraduate School, Monterey, CA 93943-5000, December 1974 (Revised September 1976).
5. Forrest, R. N., "Notes on Search, Detection and Localization Modeling," NPS71-87-001 (Fourth Revision), Naval Postgraduate School, Monterey, CA 93943-5000, April 1987 (Revised October 1990).
6. Forrest, R. N., "Three Position Estimation Procedures," NPS55-84-13 (Second Revision), Naval Postgraduate School, Monterey, CA 93943-5000, June 1984 (Revised April 1989).
7. Thompson, K. P. , "Position-Fixing and Position-Predicting Programs for the Hewlett-Packard Model 9830A Programmable Calculator," NRL Memorandum Report 3265, Naval Research Laboratory, Washington, DC , April 1976.
8. Pollock, S. M., "Search Detection and Subsequent Actions: Some Problems on the Interfaces," Operations Research, Vol. 19, No. 3, pp 559-586, 1971.

Initial Distribution List

	Copies
Research Office (Code 08) Naval Postgraduate School Monterey CA 93943-5000	1
Defense Technical Information Center Cameron Station Alexandria VA 22314	2
Library (Code 524) Naval Postgraduate School Monterey CA 93943-5000	2
Naval Postgraduate School Monterey CA 93943-5000 Code OR/Fo	15

DUDLEY KNOX LIBRARY



3 2768 00327802 9